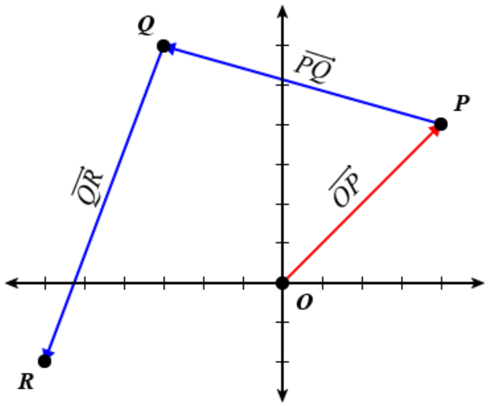
Directed Line Segments

Definition: The directed line segment from a point P in $\mathbb{R}^2$ to a point Q in $\mathbb{R}^2$ is drawn as an arrow with starting point P and tip Q. It is denoted by $\vec{PQ}$.

Example

Let $P = (4, 4)$, $Q = (-3, 6)$, $R = (-6, -2)$, and $O = (0, 0)$.



Note: Points are represented by capital letters, while vectors are represented by lowercase letters. We will be switching back and forth between the two.

Directed Line Segments

Definition: A directed line segment that starts at the origin and ends at a point P is called the position vector for P.

To describe a point in $\mathbb{R}^2$ you need two pieces of information:

- its distance from the origin,
- and which direction to travel that distance.

Definition: We define two directed line segments $\vec{PQ}$ and $\vec{RS}$ to be equivalent if $\vec{q} - \vec{p} = \vec{s} - \vec{r}$, in which case we shall write $\vec{PQ} = \vec{RS}$. In the case where $R = O$, we get that $\vec{PQ}$ is equivalent to $\vec{OS}$ if $\vec{q} - \vec{p} = \vec{s}$.

Directed Line Segments

Example

Find a vector equation of the line through $P = (4, 4)$ and $Q = (-3, 6)$.

Solution

$\vec{PQ}$ is a direction vector for the line.

$\vec{PQ} = \vec{q} - \vec{p} = \begin{bmatrix} -3 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$

So the line goes through the point $(4, 4)$ with direction vector $\begin{bmatrix} -7 \\ 2 \end{bmatrix}$.

This gives a vector equation $\vec{x} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} + t\begin{bmatrix} -7 \\ 2 \end{bmatrix}, t \in \mathbb{R}$.

The line also goes through the point $(-3, 6)$, so we could write a vector equation $\vec{x} = \begin{bmatrix} -3 \\ 6 \end{bmatrix} + t\begin{bmatrix} -7 \\ 2 \end{bmatrix}, t \in \mathbb{R}$.

$\vec{QP}$ is another direction vector for the line.

$\vec{QP} = \vec{p} - \vec{q} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} - \begin{bmatrix} -3 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$

Using this direction vector with the point $P = (4, 4)$ we get vector equation $\vec{x} = \begin{bmatrix} 4 \\ 4 \end{bmatrix} + t\begin{bmatrix} 7 \\ -2 \end{bmatrix}, t \in \mathbb{R}$.

If we use the point $Q = (-3, 6)$ with this direction vector we get vector equation $\vec{x} = \begin{bmatrix} -3 \\ 6 \end{bmatrix} + t\begin{bmatrix} 7 \\ -2 \end{bmatrix}, t \in \mathbb{R}$.

Directed Line Segments

Example

Let L_1 be the line through $O = (0, 0)$ and $P = (4, 4)$, and let L_2 be the line through $Q = (-3, 6)$ and $R = (-6, -2)$. Are L_1 and L_2 parallel?

Solution

We only need to know if $\overrightarrow{OP} = t\overrightarrow{QR}$ for some $t \in \mathbb{R}$.

$$\overrightarrow{OP} = \vec{p} - \vec{0} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\overrightarrow{QR} = \vec{r} - \vec{q} = \begin{bmatrix} -6 \\ -2 \end{bmatrix} - \begin{bmatrix} -3 \\ 6 \end{bmatrix} = \begin{bmatrix} -3 \\ -8 \end{bmatrix}$$

Since $\overrightarrow{QR}$ is not a scalar multiple of $\overrightarrow{OP}$, the lines are not parallel.