Finding the Normal to a Plane

Another feature of the cross product, $\vec{u} \times \vec{v}$, is that it is orthogonal to both $\vec{u}$ and $\vec{v}$.
So, if $\vec{u}$ and $\vec{v}$ are linearly independent direction vectors for a plane, then $\vec{u} \times \vec{v}$ is a normal vector that we look for when we are trying to find a scalar equation for the plane.

Example

Find the scalar equation for a plane with vector equation $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} + t_1 \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} + t_2 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.

Solution

First we need to find the normal vector for the plane.

$$\vec{n} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} (0)(2) - (3)(2) \\ (3)(1) - (2)(2) \\ (2)(2) - (0)(1) \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \\ 4 \end{bmatrix}$$

So, we know that the scalar equation of the plane has the form $-6x_1 - x_2 + 4x_3 = d$.
Then we plug in the point $(1, -1, 4)$ on the plane to get $-6(1) - (-1) + 4(4) = 11$.
Thus the scalar equation of the plane is $-6x_1 - x_2 + 4x_3 = 11$.