

Rank Theorem

For an $m \times n$ matrix A , the rank of A = the number of leading 1s in the reduced row echelon form of A

= the number of non-zero rows in any row echelon form of A

= $\dim(\text{Row}(A))$

= $\dim(\text{Col}(A))$

= $n - \dim(\text{Null}(A))$

Notice that $\dim(\text{Row}(A)) = \dim(\text{Col}(A))$.

This does not mean that $\text{Row}(A) = \text{Col}(A)$.

The last fact comes from Theorem 3.4.7, which says that $\dim(\text{Null}(A)) = n - \text{rank}(A)$.

Theorem 3.4.8 - Rank Theorem

If A is any $m \times n$ matrix, then

$$\text{rank}(A) + \text{nullity}(A) = n$$

This is also referred to as the Rank-Nullity Theorem.