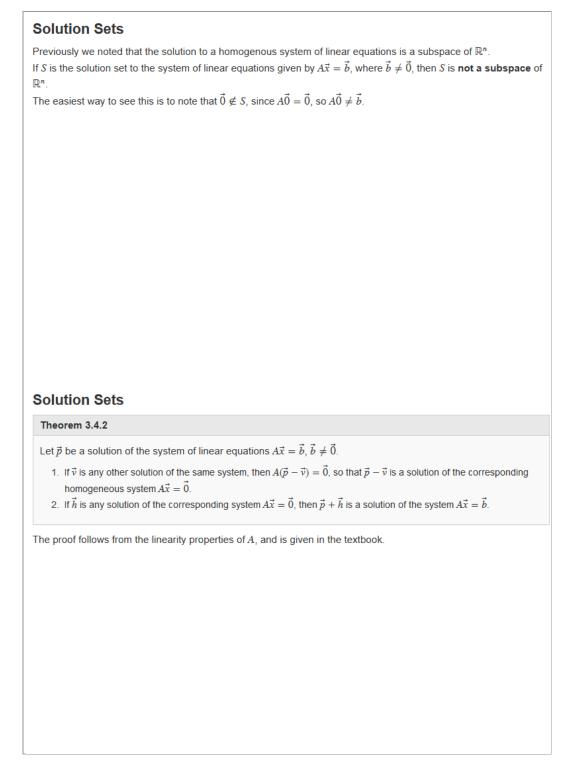
# **MATH 106** MODULE 3 LECTURE n COURSE SLIDES

(Last Updated: April 17, 2013)



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(Last Updated: April 17, 2013)

### **Solution Sets**

#### Theorem 3.4.2 - Addition

Let  $\vec{p}$  be a solution of the system of linear equations  $A\vec{x} = \vec{b}$ ,  $\vec{b} \neq \vec{0}$ , and let Null(A) be the solution set for the corresponding homogeneous system  $A\vec{x} = \vec{0}$ . Then the solution set for  $A\vec{x} = \vec{b}$  is

$$S = \{\vec{x} + \vec{p} \mid \vec{x} \in \text{Null}(A)\}$$

## Proof

To prove this fact, we need to show two things:

- 1. Every element of S (that is, every vector of the form  $\vec{x} + \vec{p}$  where  $\vec{x} \in \text{Null}(A)$ ) is a solution of  $A\vec{x} = \vec{b}$ , and
- 2. Every solution to  $A\vec{x} = \vec{b}$  is in S, that is of the form  $\vec{y} + \vec{p}$  where  $\vec{y} \in \text{Null}(A)$ .

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- (1.) follows directly from part (2.) of Theorem 3.4.2.

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- (1.) follows directly from part (2.) of Theorem 3.4.2.

For (2.), suppose  $\vec{h}$  is a solution to  $A\vec{x} = \vec{b}$ .

Then Theorem 3.4.2 part (1.) says that  $\vec{h} - \vec{p} \in \text{Null}(A)$ .

So, let  $\vec{h} - \vec{p}$  be our " $\vec{y}$ ", and we have that  $\vec{h} = (\vec{h} - \vec{p}) + \vec{p} = \vec{y} + \vec{p}$ , where  $\vec{y} = (\vec{h} - \vec{p}) \in \text{Null}(A)$ .

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The addition to Theorem 3.4.2 says that the solution set of a non-homogeneous system is simply a translation of the solution set of the homogeneous system.

So, if the solution to the homogeneous system was a line through the origin, then the solution to the non-homogeneous system is a line through  $\vec{p}$ .

If the solution to the homogeneous system was a hyperplane through the origin, then the solution to the non-homogeneous system is a hyperplane through  $\vec{p}$ .

It also means that if you know that the general solution to  $A\vec{x} = \vec{0}$  is  $Span\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ , then the general solution to  $A\vec{x} = \vec{b}$  would be  $t_1\vec{v}_1 + t_2\vec{v}_2 + \cdots + t_n\vec{v}_n + \vec{p}$ .