Systems of Linear Equations

An equation is considered linear if it doesn't involve multiplying variables.

Examples

$2x + 3y = 5$ is a linear equation.
$2xy = 5, 2x^2 + 3y = 5$, and $2x + 3\sqrt{y} = 5$ are **not** linear equations.

Note: Use of the word linear in this course refers to linear combinations, not lines.

Definition: A linear equation in n variables $x_1, \ldots, x_n$ is an equation that can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

The numbers $a_1, \ldots, a_n$ are called the coefficients of the equation, and b is usually referred to as the right-hand side, or the constant term. The x_i are the unknowns or variables to be solved for.

Systems of Linear Equations

Definition: A vector $\begin{bmatrix} s_1 \\ \vdots \\ s_n \end{bmatrix}$ in $\mathbb{R}^n$ is called a solution of a linear equation if the equation is satisfied when we make the substitution $x_1 = s_1, x_2 = s_2, \ldots, x_n = s_n$.

Example

$\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$ is a solution to $4x_1 + 3x_2 + 2x_3 = 9$ because $4(2) + 3(-3) + 2(5) = 9$.

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Definition: A general system of m linear equations in n variables is written in the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

Note: The coefficient a_{ij} is in the i-th equation, with the j-th variable. If you recall that the last coefficient is a_{mn}, not a_{nm}, it will be easier to remember that number representing the equation comes first, and the number representing the variable comes second.

Systems of Linear Equations

Definition: The solution set to a system of linear equations is the collection of all vectors that are solutions to all the equations in the system. This set will be a subset of $\mathbb{R}^n$, but it may be the empty set.

Example

The solution set for the system

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_1 + x_2 &= -1 \end{aligned}$$

is $\varnothing$ since we can never simultaneously have $x_1 + x_2 = 1$ **and** $x_1 + x_2 = -1$.

Example

The solution set for the system

$$\begin{aligned} x_1 &= 1 \\ x_1 + x_2 &= 3 \\ x_1 + x_2 + x_3 &= 6 \end{aligned}$$

is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$.

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