Vectors in $\mathbb{R}^3$

Definition: $\mathbb{R}^3$ is the set of all vectors of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, where x_1, x_2 and x_3 are real numbers. In set notation we write

$$\mathbb{R}^3 = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid x_1, x_2, x_3 \in \mathbb{R} \right\}$$

The values x_1, x_2, x_3 are called the components of the vector $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, with x_1 as the first component of $\vec{x}$, x_2 as the second component of $\vec{x}$, and x_3 is the third component of $\vec{x}$.

We equate the vector $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ with the point (x_1, x_2, x_3) and the directed line segment from the origin to the point (x_1, x_2, x_3).

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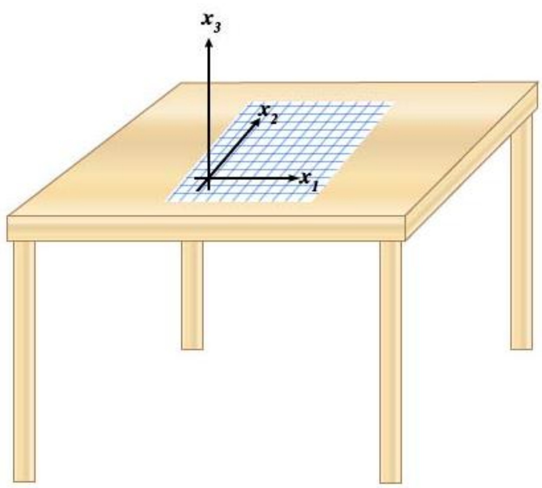

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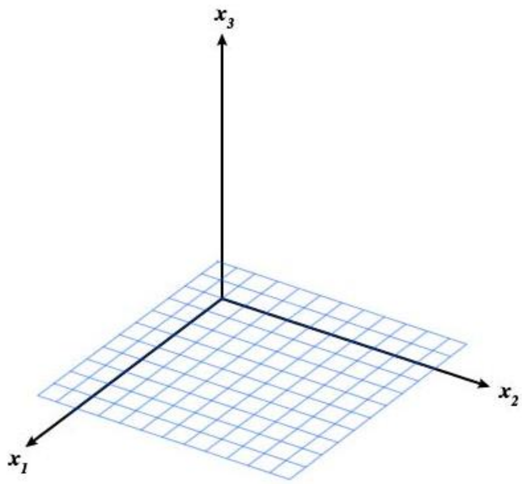

Vectors in $\mathbb{R}^3$

Definition: If $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \in \mathbb{R}^3$, $t \in \mathbb{R}$, then we define addition of vectors by

$$\vec{x} + \vec{y} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{bmatrix}$$

and we define scalar multiplication of the vector $\vec{x}$ by the scalar t by

$$t\vec{x} = t\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} tx_1 \\ tx_2 \\ tx_3 \end{bmatrix}$$

Examples

$$\begin{bmatrix} 2 \\ 7 \\ -5 \end{bmatrix} + \begin{bmatrix} 1 \\ -12 \\ 8 \end{bmatrix} = \begin{bmatrix} 2+1 \\ 7-12 \\ -5+8 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 3 \end{bmatrix}$$

$$-5\begin{bmatrix} 4 \\ -10 \\ -3 \end{bmatrix} = \begin{bmatrix} (-5)(4) \\ (-5)(-10) \\ (-5)(-3) \end{bmatrix} = \begin{bmatrix} -20 \\ 50 \\ 15 \end{bmatrix}$$

The parallelogram rule (or end-to-end rule) for addition of vectors still holds in $\mathbb{R}^3$.

Notation

- $-\vec{x} = (-1)\vec{x}$.
- $\vec{x} - \vec{y} = \vec{x} + (-1)\vec{y}$.
- The directed line segment from P to Q will be notated $\vec{PQ}$, and $\vec{PQ} = \vec{q} - \vec{p}$.

Example

Consider the points $P(1,5,7)$, $Q(2,4,3)$ and $R(3,3,-1)$ in $\mathbb{R}^3$.

They correspond to the vectors $\vec{p} = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$, $\vec{q} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$ and $\vec{r} = \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix}$.

We have $\vec{PQ} = \vec{q} - \vec{p} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$.

$\vec{RQ} = \vec{q} - \vec{r} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$.

And so we see that $\vec{PQ} = -\vec{RQ}$.

Notation

- $-\vec{x} = (-1)\vec{x}$.
- $\vec{x} - \vec{y} = \vec{x} + (-1)\vec{y}$.
- The directed line segment from P to Q will be notated $\vec{PQ}$, and $\vec{PQ} = \vec{q} - \vec{p}$.
- $\vec{0}$ is now equal to $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.
- $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $\vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Example

Let $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\vec{y} = \begin{bmatrix} 4 \\ -2 \\ 5 \end{bmatrix}$.

Then $2\vec{x} + 5\vec{e}_1 = 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$.

The equation $2\vec{y} + 5\vec{e}_1 = 2\begin{bmatrix} 4 \\ -2 \\ 5 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 13 \\ -4 \\ 10 \end{bmatrix}$.